

# **Bohr's Reply to EPR**

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[Part of joint project with Elise Crull]

## **Introduction**

Bohr's reply to EPR is arguably one of his most widely read papers directly concerned with interpretation.

It has also prompted a lot of debate and confusion in the secondary literature (at least in the philosophy literature): logical structure, role of mechanical disturbance, role of reference frames, Bohr's (alleged) positivism, etc.

Aim: to help clear some of the confusion in the philosophy literature, and hopefully gain insights into Bohr's views, especially on complementarity (work in progress!).

In particular, we shall look at (some of) the following points:

- logical structure
- contrast to the classical case
- theory of measurement
- reaction on the measuring instruments
- physical reality

If time allows: deep analogy with the treatment of the Heisenberg microscope by Grete Hermann (March 1935).

## On the text

Bohr's reply extant in the published version, and in two typescripts in the Bohr Archive: one — identical to the published version — in Bohr's English, and one in rather better German (with handwritten corrections *not* in Bohr's hand).

At least one previous draft was circulated (Schrödinger quotes from it in his letter to Bohr of 13 October 1935!), but is neither in the Bohr Archive nor in the Schrödinger *Nachlass*.

## Bohr's introductory sketch

After describing the EPR criterion of reality, Bohr writes:

By means of an interesting example, to which we shall return below, [EPR] next proceed to show that in quantum mechanics, just as in classical mechanics, it is possible under suitable conditions to predict the value of any given variable pertaining to the description of a mechanical system from measurements performed entirely on other systems which previously have been in interaction with the system under investigation.

Applying the criterion of reality, it follows in particular that canonically conjugate quantities must have definite values, which shows quantum mechanics to be incomplete.

In the quoted passage, we take it Bohr actually *agrees* with EPR.

Indeed, he says this is merely a special case of the general description of measurements in quantum mechanics.

(We shall spend some time discussing the latter.)

We take it he objects to the application of the criterion of reality.

According to Bohr, the problem with the argument is that the ‘*finite interaction between object and measuring agencies*’ (original emphasis) entails ‘a final renunciation of the the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality’.

And this is so ‘because of the impossibility of controlling the reaction of the object *on the measuring instruments* if these are to serve their purpose’ (emphasis added), and because ‘[i]n fact’, the EPR criterion of reality ‘contains [...] an essential ambiguity when it is applied to the actual problems with which we are here concerned’.

## **Logical structure**

How to block the application of the criterion of reality by invoking an ambiguity in the criterion of reality?

We suggest that Bohr is understanding the logic of EPR as follows.

(Let ‘predict a value’ be short for ‘predict a value with certainty without disturbing the system’, and ‘reality’ be short for ‘a corresponding element of reality’.)



Bohr's understanding of EPR (as we see it):

(1)  $\diamond(\text{predict a value for } Q_2) \rightarrow \exists(\text{reality for } Q_2)$

(2)  $\diamond(\text{predict a value for } P_2) \rightarrow \exists(\text{reality for } P_2)$

(3)  $\diamond(\text{predict a value for } Q_2)$

(4)  $\diamond(\text{predict a value for } P_2)$

Premises (1) and (2) follow from the EPR criterion, and (3) and (4) from the EPR example. Therefore:

$\exists(\text{reality for } Q_2) \wedge \exists(\text{reality for } P_2)$

Bohr explicitly disambiguates between these two senses of ‘without disturbing the system’:

- ‘without mechanically disturbing the system’
- ‘without disturbing the conditions affecting the possibility of predictions on the system’

We suggest to interpret his logic as follows:

- If it means ‘without disturbing the conditions affecting the possibility of predictions on the system’, then one can presumably accept (1) and (2), as in the classical case, but one should reject (3) and (4).
- If ‘without disturbing the system’ signifies ‘without mechanically disturbing the system’, one can accept premises (3) and (4), but one should reject (1) and (2).

In either case, two out of four of EPR’s premises are blocked.

Thus, Bohr needs to argue:

- (a) that in predicting  $Q_2$  or  $P_2$  we in fact ‘disturb the conditions affecting the possibility of predictions on the system’,
- (b) that in this case the reality of  $Q_2$  and of  $P_2$  is not warranted.

## Classical case

Let two bodies S and A interact (e.g. they collide).

Let the total energy and momentum be known *after* the collision.

(E.g. we knew initial energy and momentum of both, we know they have collided elastically, but we do not know their shapes.)

Measuring the energy of A, we can predict that of S.

Measuring the momentum of A, we can predict that of S.

*Remarks:*

The interaction *may well* disturb S, although there is no disturbance during ‘the last critical stage of the measuring procedure’. So it is irrelevant that in classical mechanics one can in principle implement measurement interactions that do not disturb the system at all.

Energy and momentum of A could of course be measured simultaneously, but the argument *does not use this fact*. So it is irrelevant that classical and quantum mechanics are different on this point.

Finally, S and A need not have interacted directly: e.g. let them pass through two slits in a movable screen (with known initial energy and momentum), colliding elastically with the screen as they do so.

If we know the initial energy and momentum of S and of A, and if we measure the final energy and momentum of the screen, we have prepared the same state as above.

(Note the analogy with Bohr's method of preparing the EPR state!)

## **Description of measurement**

Bohr's main example is of a particle passing through an initially movable screen with a single slit.

The initial momentum of the particle and of the screen are known.

Passage through the screen narrows down the particle's position, and particle and screen 'exchange momentum'.



After this premeasurement phase (which possibly involves collapse), the sum of the momenta is known, as is the difference in position between the particle and (the hole in) the screen.

The analogy with the EPR state should now be evident.

(Later Bohr describes also how to prepare two microscopic particles in the EPR state, but the main example already suffices.)

But now one is still *free to choose* how to complete the measurement ('both times *without* disturbing the particle mechanically', Pauli to Schrödinger, 9 July 1935):

- measuring the position of the screen immediately after the passage allows one to predict the position of the particle (for the case where an immediately subsequent position measurement on the particle is performed);
- measuring the momentum of the screen after passage allows one to predict the momentum of the particle (for the case where a subsequent momentum measurement on the particle is performed).

*Remarks:*

Bohr conceives of a measurement as involving a system  $S$ , an ancilla  $A$ , and a projective measurement on the ancilla (this is a rather modern conception!).

The example is not quite general (was he aware of this?), because the state after the premeasurement is maximally entangled (at least approximately — the latter is true also in the case of the Heisenberg microscope).

Bohr thinks of a measurement in terms of preparation (the initial state of the particle is known!).

*Crucial remark:*

Bohr's model does not seem to be a 'disturbance model' of measurement.

In the first (premeasurement) stage we generally do have disturbance, but that is true also classically!

In the second (measurement) stage instead we no longer affect the system, but only interact with the ancilla.

The disanalogy with the classical case seems to come about only with the uncontrollable reaction of the system *on the ancilla*.

Is he shifting away from a (previous) disturbance model?

Not sure:

- He seems to present this as something well known.
- The reaction of the system on the ancilla is hinted at already in the Como lecture.
- Also Pauli and even Schrödinger appear to appreciate the analogy between measurements and EPR, possibly even independently of Bohr's reply.

Pauli to Schrödinger, 9 July 1935:

A pure case of  $A$  is an overall situation in which the results of particular measurements on  $A$  (a maximal set) are predictable with certainty. I have nothing against calling this the ‘state’ — but even then it *is* the case that changing the state of  $A$  — i.e. that which is predictable of  $A$  — lies within the *free choice* of the experimenter even without directly disturbing  $A$  itself — i.e. even *after* isolating  $A$ .

And (from the same letter):

In my opinion *there is in fact no problem here* — and one knows the fact in question even without the Einstein example.

Schrödinger to Teller, 14 June 1935:

According to quantum mechanics, the preparation of a system, whereby it is brought into a certain given state, does not merely consist in material treatment of the system with tools of all kinds, but, rather, what happens afterwards depends on what one does with the tools – whether one burns them, melts them down, tramples on them or preserves them in a museum – but in particular whether one pays attention to the signs of wear on the tools, and which ones.

And (from the same letter):

This assumption arises from the standpoint of the savage, who believes that he can harm his enemy by piercing the enemy's image with a needle.

## Reaction on the measurement instruments

As mentioned, Bohr needs to argue in particular

- (a) that in predicting  $Q_2$  or  $P_2$  we in fact ‘disturb the conditions affecting the possibility of predictions on the system’.

This seems to be related to the ‘impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose’, as follows.



In case we want to use the screen to predict the particle's *momentum*, we must track the way the particle affected the screen's momentum when it passed through the slit.

But Bohr tells us that, in fact, in order to use the screen to predict the particle's *position*, we have to bolt it to the rigid support that defines the frame of reference, and we *cut ourselves off* from any possibility of tracking the uncontrollable exchange of momentum between particle and screen.

Similarly, in case we want to use the screen to predict the particle's *position*, we must track the way the particle affected the screen's position when it passed through the slit.

But, in fact, in order to use the screen to predict the particle's *momentum*, we must determine the momentum of the screen, thereby treating its position as completely uncertain, and we *cut ourselves off* from any possibility of tracking the uncontrollable displacement of particle and screen.

This is explicitly stated as an instance of complementarity.

In one case we can apply the ‘causal picture’ (conservation of momentum).

In the other case we can apply the ‘space-time picture’.

Thus far, the reconstruction seems fairly straightforward.

But what does Bohr mean with ‘cutting ourselves off’?

*A red herring?*

Assume the screen exchanges momentum with the frame of reference, so that now the momentum of the particle with respect to the frame of reference has changed.

However, we cannot track this change, because the frame of reference itself is what *defines* momentum.

This, we have once and for all renounced the possibility of taking this momentum transfer into account.

Suggestive, because a strongly *physical* explanation for why a measurement of the screen would not disturb the particle mechanically, but indeed affect the *conditions under which its momentum is well-defined*.

Among recent commentators, Dickson takes this up, and elaborates it in the language of quantum reference frames as a justification for the uncertainty principle.

On the other hand, he questions whether Bohr's use of the reference frame is too tied to the specific example of the EPR state, and fails to generalise.

Also Schrödinger seems to read the argument thus, writing to Bohr on 13 October 1935, but he points out a snag:

[P]erhaps making a very precise coordinate or momentum measurement of particle No. 2 *from the coordinate frame* (standing on it, so to speak) does not after all have an arbitrarily small effect on the coordinate frame, no matter how heavy and massive and solid the coordinate frame might be. But this [...] in fact does not apply. For had we [...] first determined the situation  $S'$  of particle No. 1 through a *direct* measurement, we would not believe that this  $S'$  could be modified through further measurements that we make from the same iron platform on other light particles, e.g. also on particle No. 2. The physical influence of such a measurement on the platform is therefore considered to be negligible.

What Schrödinger is saying is that, on this reading, Bohr's argument shows too much: it would apply to any other system *whether or not* it is entangled with the measured particle.

But if we first measure particle 1 (the distant particle), we disentangle the state of the two particles, and quantum mechanics predicts *no* further effect of the measurement of the nearby particle on the state of the distant particle!

A charitable interpretation of Bohr is to be preferred.

Indeed, it is hard to believe he failed to see his alleged mistake after Schrödinger's letter:

I believe that my response to Einstein's article has already come out, and as you will see, I have on multiple points attempted to demonstrate the thinking perhaps more clearly, and I also hope that your first objection mentioning the measurement setup was thereby answered. (Bohr to Schrödinger, 26 October 1935)

(It would be interesting, however, to locate the earlier draft and check it on this point!)



*A real herring?*

Howard on Bohr's doctrine of classical concepts (1):

Bohr recognised that the 'uncontrollable exchange of quanta of action' destroys separability of system and apparatus, which is *necessary* for the objectivity of a measurement.

(This can be formally represented as the entanglement of system and apparatus.)

Howard on Bohr's doctrine of classical concepts (2):

This objectivity is regained in each observational context. In defining a context, however, Bohr does not consider the apparatus classical and the system quantum mechanical, but treats *certain aspects* of the apparatus *as well as* of the system classically, renouncing a classical description of other aspects.

(This can be formally represented as substituting for the entangled state an appropriate mixture.)

[N.B. This formal representation is an aid to our intuition, not necessarily what Bohr himself would have said.]

Our reading fits perfectly with Howard's treatment.

Indeed, taking a projective measurement on the ancilla, or just appropriately interacting with the ancilla (bolting the screen to the support) *turns the EPR state into an appropriate mixture.*

This allows us to *selectively exploit the correlations* of the EPR state:

- *either* the correlations in position ('applying the space-time picture'),
- *or* the correlations in momentum ('applying the conservation theorem').

## Reality

To complete his reply, Bohr needs to argue

- (b) that if in making predictions with certainty we ‘disturb the conditions affecting the possibility of predictions on the system’, then the reality of  $Q_2$  and of  $P_2$  is not warranted.

Bohr is notoriously at his least explicit here, stating that

these conditions constitute an inherent element of the description of any phenomenon to which the term physical reality can be properly attached.

And also that

we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way.

The point may be simply (?) that complementarity shows that such properties as may be predicted of systems are ill-defined in general.

But maybe there is an implicit point about locality: the conditions under which things can be predicted of systems can be influenced *without* influencing the system.

In particular, we can make, say, momentum *in principle* unpredictable by manipulating just the ancilla.

Classically, we may be unable in practice to keep track of momentum exchanges, but we never *in principle* cut ourselves off from the possibility of doing so.

The fact that we can do so in quantum mechanics may after all cast doubt on the reality of quantities on the system after it has become entangled with the ancilla.

One might want to introduce ‘Bohr’s necessary criterion of reality’:

$$\exists(\text{reality for } X) \rightarrow \square(X \text{ predictable in principle})$$

where ‘predictable in principle’ means something akin to ‘predictable in principle in those circumstances in which the system is not mechanically disturbed’!



Any positivist overtones here? Or any new reading of Bohrs alleged positivism?

Does not seem straightforwardly verificationist.

Perhaps more of the kind of ‘the theory tells you what can be observed’.

(Cf. Heisenberg’ recollections about his conversation with Einstein and the genesis of the uncertainty paper: for Heisenberg, this is Einstein’s *repudiation* of positivism.)

Bohr's argument is directed at the published version of EPR. Does it apply at all to Einstein's own version?

Note that Pauli saw through the *Gelehrsamkeit* of the published version, understood Einstein's core argument, and *did* think Bohr's reply adequately addressed it (cf. Pauli to Heisenberg 15 June 1935).

## **Hermann and Bohr**

Shine further light on Bohr's reply by looking at Grete Hermann's treatment of the Heisenberg microscope.

The similarities will suggest taking Hermann as a key for reading certain aspects of Bohr.

*Hermann on the Heisenberg microscope:*

Take an electron in a plane (position within the plane completely uncertain, momentum in the plane known — approximately so, because the microscope is finite).

Illuminate with a  $\gamma$ -ray photon of known momentum.

[N.B. Analogy with EPR: immediately after the collision, sum of momenta known and difference of positions zero.]

Place a photographic plate:

- (a) in the image plane of the microscope, or
- (b) in the focal plane, or
- (c) nowhere at all

How both conceptions [the wave picture and the particle picture] are consistent with one another depends on the type of measurement: if the light is absorbed in the image plane of the observed object, then one is to work in the wave picture with the conception of a spherical wave propagating from one point, and correspondingly to ascribe a sharp position but a smeared exchange of momentum to the corpuscularly interpreted collision between electron and light quantum. If one carries out the observation in the focal plane of the microscope, then one has to deal with a parallel beam of rays, and accordingly to work in the corpuscle picture with a precisely determined exchange of momentum but an unsharp position. The one observational context that the physicist enters through observation of the photographic plate therefore determines which features of both pictures are used.

Case (a): one can give a causal analysis of the interaction based on a selective use of aspects of the wave and the corpuscular picture.

This gives us:

- a cause for the formation of the image on the plate;
- the ability to predict the outcome for a subsequent measurement of position on the electron.

Case (b): one can give another causal analysis based on a *different* selective use of aspects of the wave and the corpuscular picture.

This gives us:

- a cause for the formation of the image on the plate;
- the ability to predict the outcome for a subsequent measurement of momentum on the electron.



Case (c): Hermann explicitly says one obtains a linear combination of product wave functions, and the photon and the electron each lack individual states, a process which is not *anschaulich*.

(N.B. This is entanglement!)

Very analogous to Bohr's discussion of particle and screen.

(With the added advantage that the subsequent position measurement need not be immediately subsequent.)

We see also this fits with Howard's analysis extremely well, both in terms of the lack of separation (entanglement) and the 'fragmented' character of the classical description.

## Three replies to EPR

Hermann, Heisenberg and Bohr all write papers in 1935 arguing for the completeness of quantum mechanics:

- Hermann before EPR,
- Heisenberg prompted by EPR but replying indirectly,
- Bohr replying directly.

All three base their analyses on essentially the same EPR-like example:

- Hermann explicitly uses the Heisenberg microscope,
- Heisenberg uses it implicitly (as he clarifies in his letter to Bohr of 19 September 1935),
- Bohr uses the single-slit setup.

However, they use different arguments for completeness:

- Hermann argues that quantum mechanics is already causally complete,
- Heisenberg that any additional variables would destroy interference effects,
- Bohr that complementarity provides a complete picture while undermining the criterion of reality.